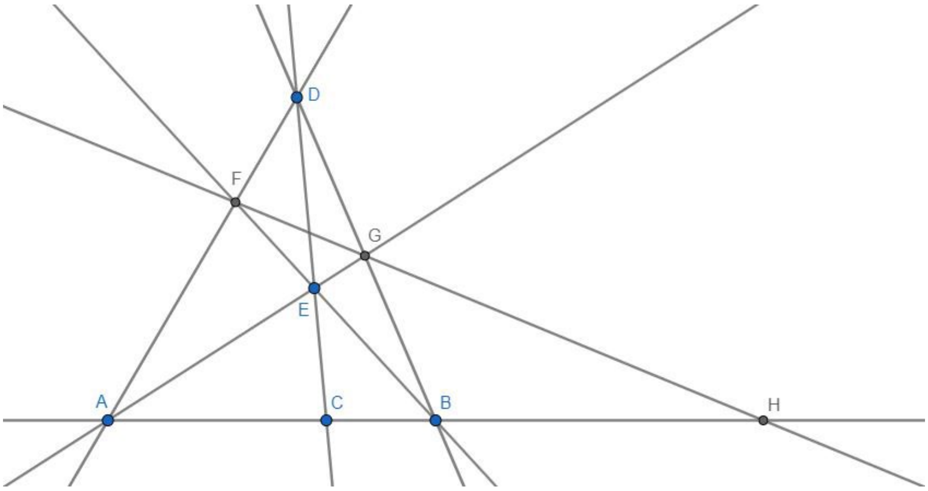
Harmonic Conjugates

Let a,b,c,d be points on a straight line. The point d is the harmonic conjugate of c with respect to a,b if the cross ratio equals -1, that is

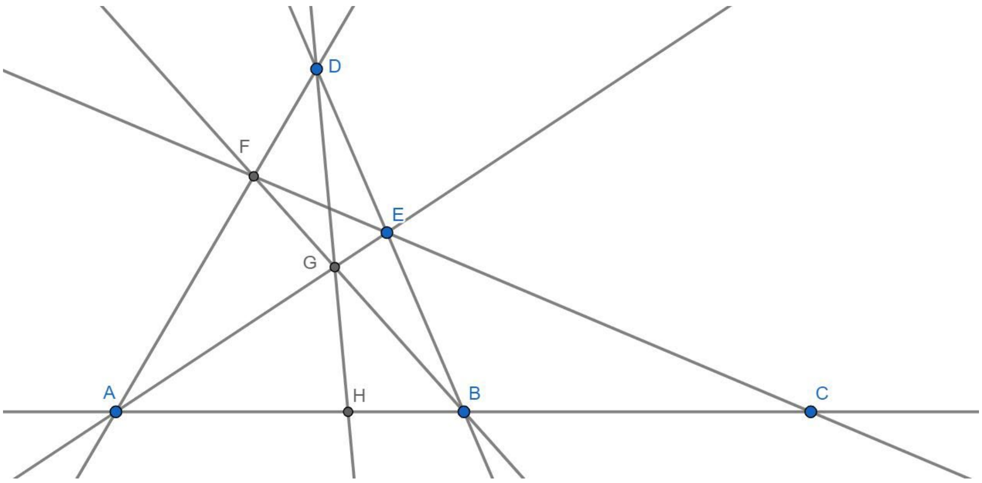
$$\frac{a-c}{a-d}\frac{b-d}{b-c}=-1.$$

By inverting each side of the equation, it is clear that c is then also a harmonic conjugate of d with respect to a,b. And, at the same time, a,b are harmonic conjugates with respect to c,d.

Harmonic conjugates shows how to geometrically construct harmonic conjugates.



Fix A,B,C as above. Choose D. Choose E on the line CD. Let F be on AD and BE and let G be on BD and AE. The point H which is on FG and AB is the harmonic conjugate of C.



In case C is not between B and C, as above, we proceed as follows. Choose D. Choose E on BD. Let F be on CE and AD and let G be on BF and AE. The point H which is on DG and AB is the harmonic conjugate of C.

Harmonic conjugates shows that the point H does not depend on the choice of D, nor on the choice of E.