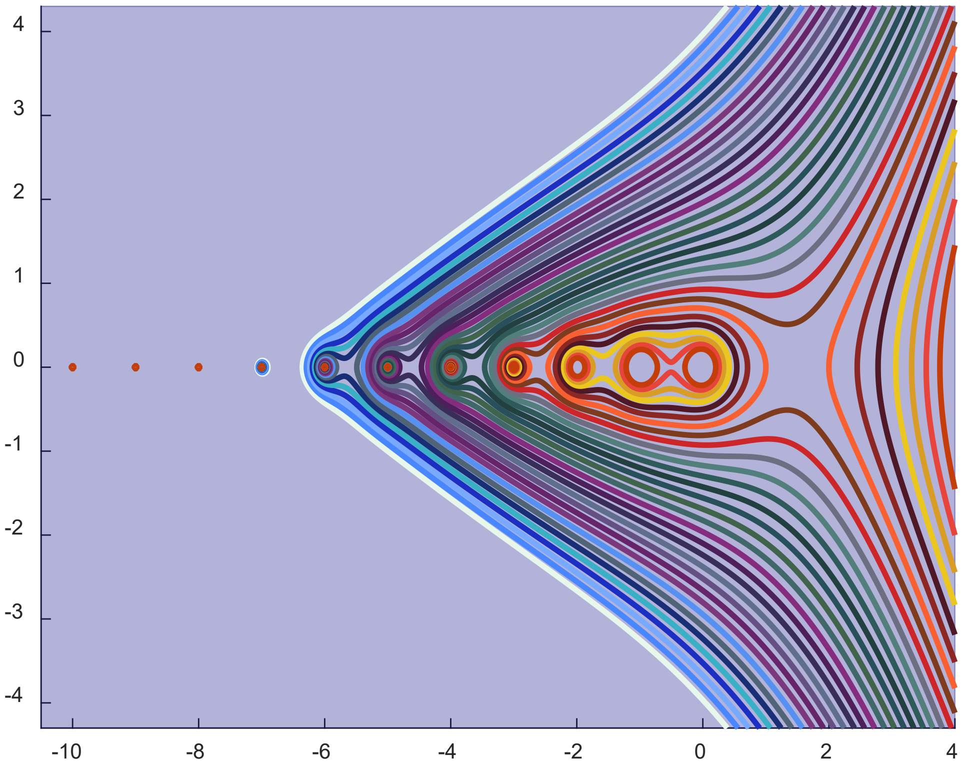
Matryoshka



When a mathematician exclaims 4! it means $4 * 3 * 2 * 1 = 24$. Have you ever wondered what $\sqrt{2}!$ would be? Or $\pi!$, or $i!$?

In 1729, in correspondence between Bernoulli, Euler and Goldbach an interpolating function of the factorials was discovered in the form of infinite products. This function, later coined the gamma function Γ by Legendre, has the property that

$$\Gamma(\mathrm{n}) = \mathrm{n}!$$

for all positive integers.

Γ has many representations, e.g., Euler also gave an integral formula. Unlike most special functions Γ does not satisfy an algebraic differential equation with rational coefficients, we say that Γ is transcendentally transcendental. Γ can be defined as the unique function with the above property which is logarithmically convex on the positive real axis.

Riemann used Γ to derive important properties of his ζ function (see The Riemann hypothesis), due to a functional relation which can be written in terms of

$$\Lambda(\mathrm{z}) = \pi^{-\frac{z}{2}}\Gamma\left(\frac{\mathrm{z}}{2}\right)\zeta(z)$$

as

$$\Lambda(\mathrm{z}) = \Lambda(\mathrm{z} - 1).$$

Matryoshka displays some level sets of the absolute value of Γ. We can see poles at zero and at the negative integers, becoming sharper and sharper.